Estimating and tracking crack growth dynamics is essential for fatigue failure prediction. A new experimental system—coupling structural and crack growth dynamics—was used to show fatigue damage accumulation is different under chaotic (i.e., deterministic) and stochastic (i.e., random) loading, even when both excitations possess the same spectral and statistical signatures. Furthermore, the conventional rain-flow counting method considerably overestimates damage in case of chaotic forcing. Important nonlinear loading characteristics, which can explain the observed discrepancies, are identified and suggested to be included as loading parameters in new macroscopic fatigue models.

Keywords: material fatigue, chaotic forcing, stochastic forcing, damage accumulation, fatigue dynamics, nonlinear characteristics

1 Introduction

The fatigue life prediction in variable amplitude loading conditions has received considerable attention, since many structures in service are not subject to constant amplitude cyclic loads. It is currently understood that load factors are nonlinearly coupled with the fatigue life [1, 2] (for example, altering the application sequence of the same large and small amplitude loads results in different fatigue lives). Thus, the most widely used method, Palmgren-Miner rule [3–5], which is based on a superposition principle, is inadequate in practice. Even, when the applied load is stationary over time, the coupling between the structural dynamics and fatigue evolution can drastically alter their dynamics [6]. Thus, it is important to identify essential nonlinear load factors that contribute to fatigue life.

In the next section, a novel fatigue testing rig based on inertial forces is described. The new testing rig has three major advantages over traditional apparatuses. In particular, it has ability to: (1) mimic complicated real-world load histories, such as chaotic and random excitations; (2) conduct fatigue tests at various $R$-ratios\(^1\) (including zero or negative), which is controlled accurately and can be adjusted during a test; and (3) conduct high frequency tests (going a little higher than 30 Hz based on the current design).

In our experiment, fatigue tests are conducted using chaotic (deterministic) and random (stochastic) load time-histories. Both loads have similar power spectral densities and probability distribution functions, but different temporal structures. The mean and variance of the excitations are kept constant throughout the experiments. Thus, superposition based cycle counting methods should be applicable. The rain-flow counting method and Palmgren-Miner rule are applied to the crack load time histories to quantify the fatigue damage variables. Nonlinear loading characteristics are proposed to explain the differences in the results, followed by a discussion and conclusions.

2 New Fatigue Testing Apparatus

2.1 Mechanical setup

The experimental system was first introduced in [7]. A picture and schematic of the test rig are shown in Fig. 1. The mechanical backbone of the system is a slip table guided by four linear bearings on two parallel rails mounted to a granite base. An LDS V722 electromagnetic shaker is used to drive the slip table under various loading conditions. The specimen is a single edge notched beam which is simply supported by pins on each end. Two inertial masses, guided by linear bearings on a central rail mounted to the slip table provide dynamic loads. The masses are kept in contact with the specimen with the use of two pneumatic cylinders. Different $R$-ratios can be obtained by adjusting the pressure

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]

where $\sigma_{\text{min}}$ is the minimum peak stress and $\sigma_{\text{max}}$ is the maximum peak stress.
The specimens are designed to follow the fracture toughness test standard ASTM E1820-08a [8] and their model is shown in Fig. 2. The specimen is made of 6061 aluminum bar stock, with dimensions 314.70 × 20.32 × 6.35 mm (length × height × width), and the fatigue is initiated by a machined ‘V’ notch. The notch is cut to a depth of 7.62 mm by a blade 0.762 mm thickness, and with a 60 degree V-shaped tip. The initial crack length was examined under a Stocker and Yale optical micrometer at 40× magnification. The crack lengths of all the fatigue tests recorded were measured to be 7.823 ± 0.069 mm.

In order to get rid of residual and machining stresses, the specimens are fully annealed prior to testing. First, the specimens are loaded into a heat treating oven, which is preheated to 775° F and remain there for two hours. Then the oven is shut off and the specimens are allowed to slowly cool. The rate of cooling is kept lower than 50° F per hour, which is achieved by the insulative properties of the oven.

2.2 Instrumentation

The structural response is measured using accelerometers and an eddy current displacement sensor. Two single-axis accelerometers from PCB Piezoelectronics, model number 333B42, are used. The sensitive axis of each accelerometer is always kept parallel with the horizontal x-axis. One eddy current sensor from Lion Precision, model number U5 with an ECL202 driver, is used to measure the relative displacement between one of the masses and the slip table.

Drive signals for the testing rig are generated by a program controlled function generator (Tektronix AFG 30222), which can generate different types of excitation signals based on either internal functions (such as random and harmonic) or stored arbitrary time histories (such as chaotic). The function generator is connected to a PC using a USB cable in order to have full control of the excitation signals. The preload is measured by two pressure sensors which are directly connected to the pneumatic cylinders. The dynamical forces between the pneumatic cylinders and the masses are measured by two piezoelectric force sensors (Model 208C02 from PCB Piezoelectronics). All data from the sensors are recorded using a data acquisition (DAQ) card from National Instrument on a PC; and all control and DAQ tasks are implemented through a LabView program.

The new design has two major advantages over a similar inertial force based fatigue apparatus described in [9, 10]. Firstly, the horizontal setup reduces the excitation force necessary to drive the system. Therefore, a similar load capacity can be achieved by using a smaller, and lower cost electromagnetic shaker. Secondly, the slip table is built using a standard mounting surface, which allows for simple mod-
ifications to accommodate various types of specimens and configurations.

3 Description of Experiment and Results

In our experiment, chaotically and stochastically excited fatigue tests are conducted. Both excitation sources exhibit similar spectral and statistical properties, but they are fundamentally different in their temporal structure. Random or stochastic source is non-deterministic, while chaotic is generated by simulating a deterministic dynamical system. In particular, we have used a steady state response of the following double-well Duffing oscillator as the chaotic excitation signal supplied to the shaker:

\[ \ddot{x} + 0.25 \dot{x} - 0.6x + x^3 = 0.2 \cos t, \]

where the over-dot indicates differentiation with respect to dimensionless time. The response of the Eq. (1) is sampled uniformly using dimensionless \( \Delta t = 0.2 \). To generate spectrally and statistically similar stochastic excitation, the deterministic Duffing signal is modified using the method of surrogate time series as described in Ref. [11]. In this iterative procedure, the chaotic signal and its spectrum’s imaginary part are randomized, and the power spectrum and the probability distribution function of the resulting stochastic signal is matched with the chaotic signal’s. Fig. 3 shows the comparison of the resulting histograms and the power spectrums of the original chaotic and the resulting stochastic signals.

The topologically equivalent phase space of the Duffing attractor is also reconstructed from the recorded \( x \) time series using delay coordinate embedding as described in Ref. [12]. In particular, a delay time of 8 samples is used to generate the Duffing phase portrait shown on the right plot in Fig. 4. The same delay was used to reconstruct the corresponding surrogate time-series phase portrait as shown in the left plot of Fig. 4, which shows the clear difference in time evolution of these trajectories: the deterministic chaotic signal has an attractor, while stochastic surrogate data does not. These two signals supplied to the shaker are henceforth called Chaotic and Random. The experimental design is very straightforward. The nominal excitation amplitude is set to keep the test time manageably short for quick turnaround time. This results in a total of ten sets of data for analysis shown in Table 1. It should be noted that all names are indicative of the order the test are run and no further inference should be made.

Some of the loads’ metrics are included in Table 1 to show that the basic statistical qualities of both signals match well and do not show any significant deviation from experiment to experiment. Any drift in these statistics, from test to test, is due to the small differences between the specimens and/or the lack of a sophisticated shaker output controller. However, these tests have proven very repeatable and the variations in the statistics of the excitation signals are negligible at this amplitude level.

The actual load histograms and power spectral densities from the slip table acceleration signals (during experiments 1-A and 1-B, respectively) are shown in Fig. 5. The acceleration signals were collected over ten minutes. The probability distributions for the voltage signals supplied to the shaker match almost identically (Fig. 3). However, these signals then pass through the shaker and couplings to reach the table. The table acceleration probability distributions are not then entirely identical as seen in Fig. 5, while the power spectral densities remain fairly similar for both stochastic and chaotic loads. Furthermore, the differences in the histograms are only observed for small amplitudes. The corresponding phase portraits are reconstructed and shown in Figure 6, where again we see the characteristics observed for the original input signals. It would be a very difficult task to build input surrogate data which would match the table acceleration
Table 1: Test results. Mean, variance, skewness and kurtosis are calculated from table acceleration data. Damage is estimated based on the rain-flow counting method and the Palmgren-Miner rule.

<table>
<thead>
<tr>
<th>Random Signal</th>
<th>TTF</th>
<th>Damage</th>
<th>Mean</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–A</td>
<td>0:53</td>
<td>0.97</td>
<td>-0.0020±0.0014</td>
<td>0.0740±0.0038</td>
<td>0.0012±0.0060</td>
<td>2.5850±0.0053</td>
</tr>
<tr>
<td>2–A</td>
<td>1:07</td>
<td>1.06</td>
<td>-0.0021±0.0014</td>
<td>0.0716±0.0038</td>
<td>0.0081±0.0049</td>
<td>2.5827±0.0066</td>
</tr>
<tr>
<td>3–A</td>
<td>0:44</td>
<td>1.00</td>
<td>-0.0020±0.0015</td>
<td>0.0742±0.0038</td>
<td>0.0092±0.0076</td>
<td>2.5980±0.0094</td>
</tr>
<tr>
<td>4–A</td>
<td>0:52</td>
<td>1.07</td>
<td>-0.0019±0.0015</td>
<td>0.0747±0.0041</td>
<td>0.0073±0.0069</td>
<td>2.5898±0.0096</td>
</tr>
<tr>
<td>5–A</td>
<td>0:45</td>
<td>0.86</td>
<td>-0.0038±0.0006</td>
<td>0.0724±0.0036</td>
<td>0.0081±0.0052</td>
<td>2.5941±0.0062</td>
</tr>
<tr>
<td>Average</td>
<td>0:52</td>
<td>0.99</td>
<td>-0.0023±0.0015</td>
<td>0.0733±0.0018</td>
<td>0.0067±0.0067</td>
<td>2.5892±0.0094</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chaotic Signal</th>
<th>TTF</th>
<th>Damage</th>
<th>Mean</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–B</td>
<td>1:44</td>
<td>1.48</td>
<td>-0.0011±0.0014</td>
<td>0.0717±0.0010</td>
<td>-0.0086±0.0052</td>
<td>2.6878±0.0770</td>
</tr>
<tr>
<td>2–B</td>
<td>1:50</td>
<td>2.08</td>
<td>-0.0031±0.0014</td>
<td>0.0717±0.0011</td>
<td>-0.0091±0.0062</td>
<td>2.6636±0.0878</td>
</tr>
<tr>
<td>3–B</td>
<td>1:46</td>
<td>1.51</td>
<td>-0.0019±0.0015</td>
<td>0.0725±0.0008</td>
<td>-0.0075±0.0068</td>
<td>2.6259±0.0775</td>
</tr>
<tr>
<td>4–B</td>
<td>1:42</td>
<td>1.39</td>
<td>-0.0025±0.0015</td>
<td>0.0721±0.0010</td>
<td>-0.0051±0.0041</td>
<td>2.6927±0.0503</td>
</tr>
<tr>
<td>5–B</td>
<td>1:13</td>
<td>1.03</td>
<td>-0.0021±0.0014</td>
<td>0.0717±0.0010</td>
<td>-0.0062±0.0050</td>
<td>2.6996±0.0699</td>
</tr>
<tr>
<td>Average</td>
<td>1:39</td>
<td>1.50</td>
<td>-0.0021±0.0016</td>
<td>0.0719±0.0010</td>
<td>-0.0074±0.0057</td>
<td>2.6720±0.0785</td>
</tr>
</tbody>
</table>

properties of the chaotic signal. As long as the main factors such as mean, and minimum and maximum acceleration are preserved, the differences are deemed acceptable.

The first and most obvious piece of data that gives an indication of a fatigue process is the time to failure (TTF). A clear trend emerges in the TTF data shown in Table 1: the random excitation is much more damaging to the specimen than the chaotic excitation. These test results show that the system under chaotic excitation outlasts the system under Random excitation by nearly a factor of two in all accounts. The question then is if these differences can be explained solely by the differences observed in the excitation histograms shown in Fig. 5. Since the mean, variance, skewness and kurtosis of the excitation do not vary significantly during the experiment, superposition principle based cycle counting methods should still account for the differences observed in the histograms.

4 Cumulative Fatigue Damage

Rain-flow counting method [13, 14] and the Palmgren-Miner rule are used here to quantify the damage based on number of stress cycles experienced. Theoretically, the criteria for failure would occur when the damage variable, \(0 \leq D \leq 1\), reaches one.

The stress at the crack tip is approximated by Euler-Bernoulli beam theory. The specimen is regarded as a simply supported beam with applied loads, as shown in Fig. 7. \(P\) is the static mean force due to the pressure within the cylinders, \(a\) is the relative acceleration between the masses and the table. In this testing, both cylinders have equal pressure. The rain-flow cycle counting method is applied to the time series of the maximum stress at the notch of the beam. The objective of this method is to reduce a variable load history signal into a sequence of constant amplitude cycles. Fig. 8 shows

Fig. 5: Histogram and power spectrum of the table acceleration data of the random (left) and chaotic (right) signals
the histograms of cycle amplitudes from stress histories of test 1-A and 1-B. While, in general, the TTF depends on the sequence in which the different amplitude cycles are applied, in our tests the load profiles are not altered over time, and the same load sequence is applied repeatedly. Thus, any differences observed in TTF are only attributable to the difference in temporal structure of each load signal.

The Palmgren-Miner rule uses an S-N curve for the test specimen to evaluate and determine the damage variable. This curve indicates how many stress cycles can a material experience at each stress amplitude level. Then the damage variable corresponding to each amplitude of stress in Fig. 8 is the ratio of the actual cycles experienced over the maximum possible as given by the S-N curve. Since \( D \) is simply a ratio, its exact value depends on the S-N curve used and is inherently nondescript. A simple S-N curve can be expressed by the equation

\[
\log_{10} S = A + B \log_{10} N,
\]

(2)

where \( S \) is the stress amplitude and \( N \) is the number of cycles until the failure at that stress level. For the comparison of chaotic and random loadings, the S-N curve is chosen to yield \( D \approx 1 \) for all random loadings. Hence, the coefficients \( A \) and \( B \) are determined to be 2.58 and \(-0.25\), respectively. Then the damage variable is calculated for each test. The results are shown in Table 1.

Typically, the Palmgren-Miner rule would be used to predict the failure by characterizing a typical stress history over some time, and approximating the stress history over the expected lifetime of the fatiguing structure. The failure criteria would be met once the damage variable \( D = 1 \). In our analysis, the damage criteria is calculated over the actual stress history of the entire load time history. Therefore, since the failure criteria is equivalent for all tests it can be expected that for all tests \( D \) would be approximately equal if the linear damage law is applicable. This is true across tests of the same type, chaotic or random. However, in comparison between the two, the linear damage law does not reflect the observed effects of the excitation signals’ time history on TTF. In particular, it significantly overestimates damage for the chaotic excitation.

The fatigue testing using random and chaotic excitation that have nearly identical statistical and spectral properties showed that specimens failed twice as fast during chaotic forcing (Table 1). The actual stress loads at the crack have almost exactly the same spectral characteristics, but histograms show that chaotic forcing has more low amplitude loads, while maintaining approximately the same loading for larger amplitudes (Figure 4). Everything being equal and keeping nominal excitation amplitude constant over the whole experiments, the linear Palmgren-Miner damage law, which is independent of the loading time-history, should have accounted for the differences in histograms. However, while it was adequate to provide consistent estimates of damage during random excitation experiments, Palmgren-Miner rule significantly overestimated the damage variable for the chaotic excitation. This result implies that the excitation’s temporal structure is very important in fatigue accumula-
where $\lambda$ is the Lyapunov exponent, and $t_n = n\Delta t$. The existence of a positive Lyapunov exponent indicates the chaotic behavior of a system. We use the algorithm used to calculate the maximal Lyapunov exponent [12, 21] to obtain feature curves that describe average divergence of nearby trajectories in the phase space over finite time period.

Since crack propagation can make our system structurally unstable [6], stress data is divided into data records of 100,000 points in each. A delay time of 7 time steps [22] is used for the delay coordinate embedding of the time series. Embedding dimension is determined for the random signal using the first minimum of false nearest neighbors (FNN) curve [24]. Zero FNN indicates complete unfolding of the attractor for that dimension, which ensures the uniqueness of the resulting reconstructed trajectory. Fig. 9 shows that an embedding dimension of 5 is optimal for both signals. However, the FFN curve never reaches zero for the random signal and continues to increase after the initial dip. While the initial dip in the FNN trend suggests the existence of short-time correlations in the random data (as in colored noise), later increase indicates its infinite dimensionality (i.e., there is no finite embedding dimension for which the embedded trajectories become unique).

The average trajectory divergence is estimated in each data record. The results for test 1-A and 1-B are shown in Fig. 10. As expected, there is a clear difference between the divergence rates observed for the chaotic and random excitations for small time scales, and these features are fairly consistent during each experiment. The divergence rate at small scale for the random signal is much higher than for the chaotic, which could also explain differences observed in histograms for the small amplitudes. For the random excitation starting at low amplitudes, large amplitudes are reached very fast. In contrast, the chaotic signal’s divergence is more gradual and slower. In other words, time correlation persists longer in the chaotic time series, which causes more coherent structures at small length and time scales in the chaotic signal.

Figure 11 shows the average trajectory divergence rates for all ten tests in the first data record. These results are consistent for each test type and again show clear difference in the random and chaotic divergence rates.

The correlation sum is usually used to estimate the fractal dimension of an attractor [23]. However, since random signal does not have a finite-dimensional attractor, we use the correlation sum as a feature differentiating our signals. Let a set of points $\{y_i\}_{i=1}^n$ uniformly sample an attractor. Then the correlation sum $C(\epsilon)$ approximates the cumulative distribution of distances on the attractor as:

$$C(\epsilon) = \frac{2}{(n-s)(n-s-1)} \sum_{i=1}^n \sum_{j=i+1}^n \Theta(\epsilon - ||y_i - y_j||)$$

and

$$\sim \epsilon^{D_2}$$

where $D_2$ is the fractal dimension of the attractor. To estimate $D_2$, we use the correlation sum $C(\epsilon)$ with different values of $\epsilon$. The fractal dimension $D_2$ is estimated by fitting the correlation sum curve to the linear function $\log(C(\epsilon))$ vs. $\log(\epsilon)$.

5 Nonlinear Characterization of Loading

The two main features of dissipative nonlinear dynamical systems are Lyapunov exponents [15–17] and fractal dimensions [18–20]. The Lyapunov exponents are the average exponential rates of separation of trajectories that are infinitesimally close initially. Fractal dimensions relate to the complexity of the system’s attractor, and can provide a lower bound on the active degrees-of-freedom in a dynamical system. These quantities are characteristics of a deterministic dynamical system, and are not defined or appropriate for stochastic systems. Here we will not focus on the estimation of the actual Lyapunov exponents or fractal dimensions, but instead look at nonlinear features extracted by applying the appropriate algorithms to the load time histories. The hope is that these features will capture the differences that can be used to explain the discrepancies in the observed TTFs.

In phase space of a dynamical system, two trajectories with initial separation $\delta_0$ diverge after $n$ time steps as

$$\delta_n = \delta_0 e^{\lambda t_n}, \quad (3)$$

where $\lambda$ is the Lyapunov exponent, and $t_n = n\Delta t$. The existence of a positive Lyapunov exponent indicates the chaotic behavior of a system. We use the algorithm used to calculate the maximal Lyapunov exponent [12, 21] to obtain feature curves that describe average divergence of nearby trajectories in the phase space over finite time period.

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and

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where $\Theta$ is a Heavyside step function, $n$ is the total number of points, $s$ is the time interval needed to remove any time correlations in the pair of points, and $D_2$ is the estimated correlation dimension. The modified algorithm given in [20] is used to estimate the correlation sum for the different length scales $\varepsilon$, and results are shown in Fig. 12. Again we see that for small length scales there are clear differences. For the chaotic signal there are considerably more smaller distances in the data than for the random signal. However, both signals have identical correlation sum for the large amplitudes or distances, which again correlates well with the similarity of the histograms for large amplitudes.

6 Discussion

Differences in excitations observed in histograms did not prove sufficient to provide consistent estimates of TTF using Palmgren-Miner rule and Rain-Flow counting method. In particular, they overestimated damage for the chaotic excitation by almost a factor of two. Additional nonlinear characteristics (trajectory divergence rates and correlation sum) showed more drastic differences in the small time- and length-scale dynamics that can explain the observed differences in TTF. Larger divergence rates for small scales in the random signal clearly correlate well with shortened TTF in the random tests. For small length- and time-scales, these larger rates cause the load on the crack to change faster for the random signal than for the chaotic. Therefore, the crack under random load experiences fewer low stress cycles when compared to the chaotic load which can explain the differences in the TTF. In particular, this suggests that fatigue life depends not only on the cycles of maximum stress experienced, but also on the rate at which this maximum stress is achieved—faster time-rate-of-change of stress being more damaging. The observed differences in divergence rates are also reflected in the deviations observed in the correlation sums at small scales. Therefore, both of these features correlate well with the observed TTF and may be used as loading parameters for a new damage model that could account for the observed differences.

7 Conclusion

In this paper, we reported on the experimental results from fatigue testing in which chaotic and random loading tests were conducted using a novel fatigue testing apparatus. Both loadings shared the same statistical and spectral properties. However, experimental results showed that random loading was more damaging than chaotic. Standard, linear Palmgren-Miner rule and rainflow counting method grossly overestimated damage for the chaotically excited system, while they were adequate to describe damage in randomly excited system. Nonlinear dynamical characteristics of the excitation time series were considered to explain the differences. These metrics clearly showed that while the signals had similar characteristics at large scale, for a small scale trajectory divergence rates were much higher in the random signal. In addition, there was more finer structure/detail in the chaotic signal as shown by the correlation sum. These observations stipulated that crack was experiencing fewer low stress cycles for the random excitation, which was also reflected in the excitation histograms. These results also suggested that fatigue life depends on the rate at which maximum stress is reached at the crack tip. Therefore, both of the considered nonlinear characteristics correlate well with the observed differences in the TTF, and may be used as loading parameters for the new damage laws providing more consistent TTF estimates.

8 Acknowledgments

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