CAN WE USE A MATLAB APPLICATION TO IMPROVE STUDENT PERFORMANCE ON TRIGONOMETRY OF COMPLEX NUMBER PROBLEM SOLVING?

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Complex numbers are a basic, interdisciplinary concept for electrical and mechanical engineering, and computer science students. To improve the performance of students in classes such as dynamics and control mechanisms, it is necessary to improve their performance on computing the real and imaginary components of complex numbers. We wrote a computer graphics, animated application in MATLAB to provide students with a method to improve their performance on computing the real and imaginary components of complex numbers. MATLAB is a good choice for doing numerical computations and GUI interface writing. We assessed one group of first-year engineering students on calculating the real and imaginary components of complex numbers before having access to the MATLAB application. We assessed the same group of first-year engineering students after access to the application. Using Student's t-test, the students' mean performance was statically no different. Interestingly, the Electromechanical majors were different from the Electrical majors at a statistical significance level of $P<0.05$.

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Introduction/Background

One of us (McCusker) teaches control theory. One way of teaching control theory is to use complex impedances both for the electronic and mechanical components of the control system. Another way is to solve the differential equations. If one uses the complex impedance method, the differential equations are changed to algebraic equations. For example, a second-order differential equation becomes a quadratic equation with either real or complex roots, therefore it is essential for the student to be able to solve a quadratic equation. Interestingly, when the damping constant of the electro-mechanical system is reduced below a critical value the roots of the quadratic equation go from real to complex, causing the behavior of the overall system to be oscillatory and possibly unstable. This is why we asked the question "can students get the relationship between the damping constant and the nature of the roots of a quadratic equation (either real or complex)." Several faculty members who teach mechanical and electrical engineering commented that students are sometimes weak in complex numbers [1,2,3]. This prompted us to do a preliminary survey of students in classes taught by one of us (Gloria Ma). We found that first-year students were weak in complex numbers. Students at University choose engineering either because they like to do it, for financial reasons or they want to contribute to humanity [4]. Preliminary data in this study and from teaching experience suggest that they have some weakness in complex numbers, which tends to discourage them from continuing in engineering. The literature on teaching complex numbers is limited to books on college algebra, and chapters of books on control theory [5]. There are many books and papers on teaching mathematics [6]. Students who received problem-based instruction for two years demonstrated significantly higher mathematics achievement than traditionally instructed students—both in problem solving and in conceptual understanding [7]. We decided to take a problem-based learning (PBL) approach to increasing student performance in trigonometry of vectors. PBL has been used to teach medicine, engineering and other subjects [8,9]. PBL refers to teaching by having students use resources such as a computer to solve problems guided by an instructor [10].

Based on this data, we did two things. First, we wrote an animated graphical application in MATLAB to try to help students improve their complex number trigonometry. Second, we surveyed students who had access to the application and used a similar group of first-year students who didn't have access to the program as a control. The reason we used MATLAB is that it is easy to program in. Due to historical reasons, the animated graphics are somewhat difficult to use. MATLAB is derived from FORTRAN [11]. MATLAB can be traced to Cleve Moler's thesis at Stanford [12]. MATLAB is one of the main products of The Mathworks which was founded in 1984. Mathworks now has approximately 1 million users [13]. Due to MATLAB's derivation from FORTRAN, it is a simple and easy to program language. Since 1984, the standard for all computers has been the graphical user interface (GUI). The GUI consists of a mouse or similar device which is used to point to buttons on the computer screen and, in our case, choose damping parameter in a quadratic equation. The roots are then calculated, either real or complex. Finally, as the damping parameter is varied, an animation shows the roots changing from real to complex in the complex plane.

The main idea of this paper is to propose an animated graphical vector simulation to improve engineering and technology students' command of the trigonometry of complex numbers.

Data/Formulation/Methodology

Starting in the spring of 2013 we tested students on their ability to calculate the horizontal component of a vector. Students were asked to use trigonometry to calculate the horizontal component of a vector "by hand" and then compare their results to an animated problem solution calculator. Animation is often used (e.g. WorkingModel software [14]) to stimulate student interest. In this preliminary study, we animated a vector rotating describing a unit circle. This testing method produced tentative results which indicated that first year students seemed to test low on their ability to calculate one component of a vector.

The main idea of this paper is to measure the improvement in first year students' performance due to the use of an animated quadratic root solution calculator. We chose to implement the solution calculator in MATLAB (see Appendix B for a flow-chart of the graphical user interface for the solution calculator).

During the 2015 fall semester, one of us (Ma) taught first-year engineering students in Introduction to Engineering. All sections of Introduction to Engineering, including more than 125 students were tested. Our tests had two parts: The first part was to test to see if the students knew basic facts about imaginary numbers for instance that i to the second power equals negative one. It also tested knowledge of higher powers of i. The second part tested whether the students understood polar notation for imaginary and complex numbers, and could do simple calculations which involved trigonometry (see Appendix A for a copy of the test). Polar notation for complex numbers is based on the idea that complex numbers can be represented as a line segment (the magnitude of the complex number) and an angle from the real axis. Polar notation is similar to vector notation for real numbers. These skills are essential in order to calculate the roots of a quadratic equation. The complex numbers calculation test was administered to these engineering students. (See Figures 1 and 2 for a summaries of their responses.) This test assessed the first-year students since, in the earlier (2013) preliminary study, they seemed the weakest group. Furthermore, we thought that the first-year engineering students were at risk for failure in dynamics and control theory since calculating complex numbers is fundamental to learning this subjects.

In the 2016 spring semester, a colleague (Douglas Sondak) administered a pre-test. Then, he exposed them to the complex root calculator application; and gave the post-test to the same students. In these tests the students were asked to calculate a vertical component (the imaginary part of a complex number), given the horizontal component (real part of the complex number) and angle of elevation of the complex numbers (see Appendix A). The tests did not record the gender or socio-economic status of the students.

Analysis

The preliminary results indicated that freshmen have some weakness in calculating complex numbers. This basic complex number skill is essential for applying complex numbers to problems in both electrical engineering and mechanical engineering. Most of our University's students have reasons other than interest in complex numbers mathematics for studying engineering (Caserta, Lind and Chedid, Spring 2011). So the low performance of freshman on complex number calculations is understandable.

First year students' performance on representing complex numbers, in polar form, was graded A,B,C,D or F. A and B were graded as correct, except that B was the grade assigned for incorrect units. D answers showed awareness of the concept of "SOH CAH TOA." A grade of F was assigned if the student gave no answer. Numerically, A=5, B=4, C=3, D=2, and F=1. The results with and without the application are shown in Fig. 1. The average with the application is 55.7. The average without the application is 51.4. In Fig. 1, we see that there is no significnt improvement for students with access to the animated graphics application. Interestingly, the Electromechanical majors were different from the Electrical majors at a statistical significance level of $P<0.05$ [15]. Please see Fig. 2.

Conclusions

More study is needed to determine the best hands on activity and software interface to teach complex number mathematics to engineering students. We suggest repeating the experiment with a lrger population of students to see if the results are consistent. Our colleague, Doug Sondak asked his students for feed-back on the complex numbers trigonometry calculator application. Students said that the application was too abstract. A new application needs to be written which solves an actual engineering problem in order to increase engineering student interest and engagement.

The authors would like to acknowledge the significant help in planning this research study rendered by Ray Nagem of Boston University, Douglas Sondak, Robert Lind and Michael Werner of our University.

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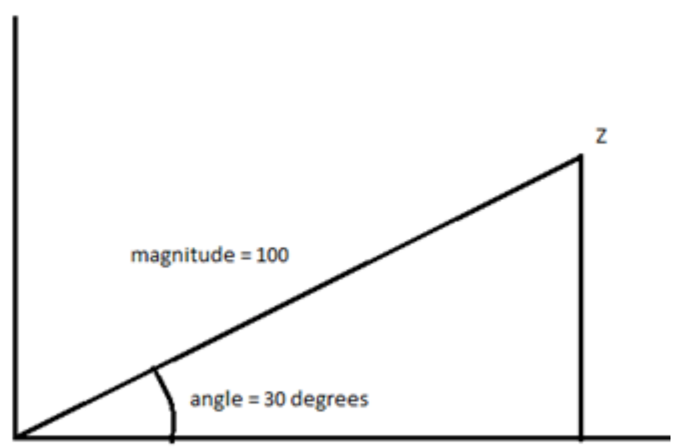
Appendix A - Test on Complex Numbers

1. Find x.
   a) $x=i^4$
   b) $x=i^5$
   c) $x=i^6$
   d) $x=i^7$
   e) $x=i^8$
2. In control theory, engineers use j as the square root of -1 instead of i.
   Find x.
   a) $x=j^4$
   b) $x=j^8$
   c) $x=j^{12}$
   d) $x=j^{14}$
   e) $x=j^{15}$
3. A complex number, z, has a magnitude of 100 and an angle of 30 degrees above the x-axis.
   Find z.



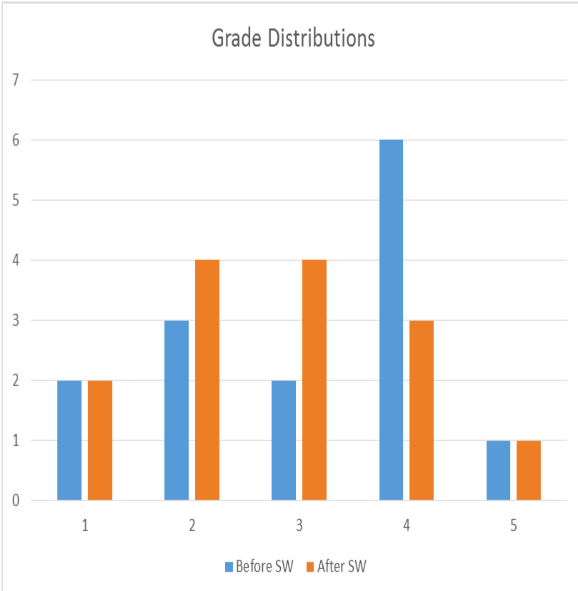


Fig. 1 Grade distributions before and after SW

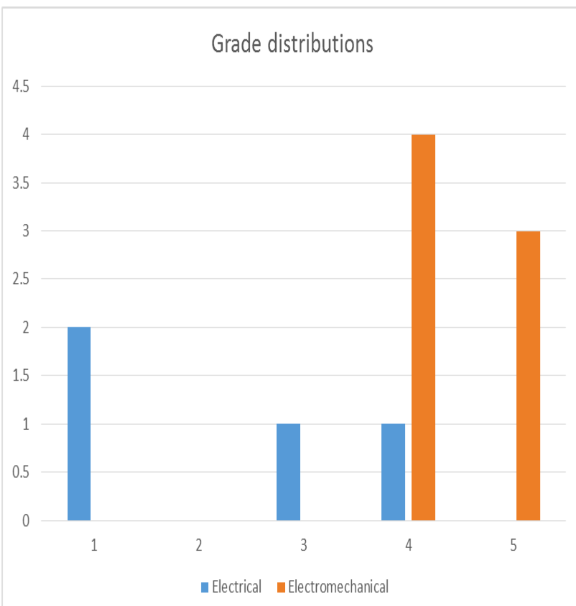


Fig. 2 Grade distributions

Appendix B - Flow Chart for the GUI